International Journal of Agriculture Sciences
ISSN: 0975-3710 & E-ISSN: 0975-9107, Volume 16, Issue 1, 2024, pp.-12901-12903.
Available online at https://bioinfopublication.org/pages/jouarchive.php?id=BPJ0000217

Research Article

RATIO TYPE ESTIMATORS FOR ESTIMATING POPULATION MEAN BY USING SOME AUXILIARY INFORMATION

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Received: December 03, 2023; Revised: January 26, 2024; Accepted: January 28, 2024; Published: January 30, 2024

Abstract: Auxiliary information plays an important role in survey sampling, enhancing the precision, accuracy, and efficiency of estimating population parameters. This study focuses on the estimation of the finite population mean through the introduction of generalized ratio-type estimators in simple random sampling without replacement. These estimators utilize the coefficient of variation and population deciles. The paper calculates and compares the expressions for mean square error and bias with those of classical and existing estimators. The comparative analysis demonstrates that our proposed class of estimators qualifies as efficient estimators based on mean square error (MSE) and percent relative efficiency (PRE) criterion. Empirical findings affirm the efficiency of our proposed estimators over classical and existing ones, as they exhibit lower mean square error and bias. Additionally, the percent relative efficiency (PRE) is consistently higher in our proposed estimators.

Keywords: *Ratio-type estimators, Coefficient of variation, Deciles, Mean square error, Bias*

Citation: Raja T.A. and Maqbool S. (2024) Ratio Type Estimators for Estimating Population Mean by Using Some Auxiliary Information. International Journal of Agriculture Sciences, ISSN: 0975-3710 & E-ISSN: 0975-9107, Volume 16, Issue 1, pp.- 12901-12903.

Academic Editor / Reviewer: Rajan Singh, Prof. R. K. Sathe

Introduction

William Gemmell Cochran (15 July 1909 – 29 March 1980) [1] was the first to introduced the classical ratio-type estimator as the method for estimating the population mean. This approach relies on utilizing prior information about an auxiliary variable X within the population. Upadhyaya & Singh (1999) [2] modified ratio type estimators using coefficient of variation and coefficient of kurtosis of the auxiliary variable. Singh *et al.,* (2004) [3] and Sisodia & Dwivedi (2012) [4] incorporated the coefficient of variation of the auxiliary variable in their studies. Subsequent enhancements in estimation techniques were accomplished through the introduction of numerous modified ratio estimators, as evidenced by the work of Subramani and Kumarpandiyan (2012) [5]. Maqbool *et al.*, (2016) [6], Subzar *et al.*, (2016, 2017, 2019) [7-9], Raja *et al.,* (2017, 2023) [10,11] utilized ratio estimators using different parameters like conventional and non-conventional parameters, Huber M estimation of auxiliary variable in sample survey.

Consider a finite population P=($P_1, P_2, P_3, \ldots, PN$) of different and identifiable units. Let be the study variable with value to be measured on giving a vector $Y=(Y_1, Y_2, Y_3, \ldots, YN)$. Here our objective is to estimate population mean $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ of a random sample.

The mean ratio estimator, formulated for the estimation of the population mean $\bar{Y}$, is defined as $\hat{\bar{Y}}_r = \frac{\bar{y}}{\bar{x}}\bar{X}$

Bias, and the mean squared error (MSE) of the ratio estimator are formulated as

$$B(\hat{\bar{Y}}_r) = \frac{(1-f)}{n}\frac{1}{\bar{X}}(RS_x^2 - \rho S_x S_Y) \quad R = \frac{\bar{Y}}{\bar{X}} \quad MSE(\hat{\bar{Y}}_r) = \frac{1-f}{n}(S_y^2 + R^2 S_x^2 - 2R\rho S_x S_Y)$$

Existing Ratio Estimators

Ratio-type estimators were proposed by Kadilar and Cingi (2004, 2006) [12,13] for the population mean in simple random sampling, incorporating known auxiliary information such as (coefficient of variation, and coefficient of correlation). Their findings demonstrated the more efficiency of the suggested estimators compared to the traditional ratio estimator when estimating the population mean.

$$\hat{\bar{Y}}_1 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{\bar{x}}\bar{X}, \; \hat{\bar{Y}}_2 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + C_x)}(\bar{X} + C_x),$$

$$\hat{\bar{Y}}_3 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + \rho)}(\bar{X} + \rho),$$

$$\hat{\bar{Y}}_4 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + \rho)}(\bar{X}C_x + \rho), \hat{\bar{Y}}_5 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + C_x)}(\bar{X}\rho + C_x),$$

Where Cx= Population coefficient of variation
ℓ= Population coefficient of correlation

Proposed Modified Ratio Estimator

Inspired by the estimators mentioned earlier, we introduce a novel class of efficient ratio-type estimators. These estimators are formulated through the linear combination of the coefficient of variation and population deciles. (D_1, D_2, D_3, D_4 D_5,D_6,D_7,D_8,D_9,D_{10}).

$$\hat{\bar{Y}}_{p1} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + D_1)}(\bar{X}C_x + D_1).$$

$$\hat{\bar{Y}}_{p2} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + D_2)}(\bar{X}C_x + D_2).$$

$$\hat{\bar{Y}}_{p3} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + D_3)}(\bar{X}C_x + D_3).$$

$$\hat{\bar{Y}}_{p4} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + D_4)}(\bar{X}C_x + D_4).$$

$$\hat{\bar{Y}}_{p5} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D_5)}(\bar{X}C_x+D_5).$$

$$\hat{\bar{Y}}_{p6} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D_6)}(\bar{X}C_x+D_6).$$

$$\hat{\bar{Y}}_{p7} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D_7)}(\bar{X}C_x+D_7).$$

$$\hat{\bar{Y}}_{p8} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D_8)}(\bar{X}C_x+D_8).$$

$$\hat{\bar{Y}}_{p9} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D_9)}(\bar{X}C_x+D_9).$$

$$\hat{\bar{Y}}_{p10} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D_{10})}(\bar{X}C_x+D_{10}).$$

The bias, related constant and the MSE for proposed estimator can be obtained as follows:

$$B(\hat{\bar{Y}}_{p1}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_1^2, \quad R_1 = \frac{\bar{X}C_x}{\bar{X}C_x+D_1} \quad MSE(\hat{\bar{Y}}_{p1}) = \frac{(1-f)}{n}(R_1^2S_x^2+S_y^2(1-\rho^2)).$$

$$B(\hat{\bar{Y}}_{p2}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_2^2, \quad R_2 = \frac{\bar{X}C_x}{\bar{X}C_x+D_2} \quad MSE(\hat{\bar{Y}}_{p2}) = \frac{(1-f)}{n}(R_2^2S_x^2+S_y^2(1-\rho^2)).$$

$$B(\hat{\bar{Y}}_{p3}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_3^2, \quad R_3 = \frac{\bar{X}C_x}{\bar{X}C_x+D_3} \quad MSE(\hat{\bar{Y}}_{p3}) = \frac{(1-f)}{n}(R_3^2S_x^2+S_y^2(1-\rho^2)).$$

$$B(\hat{\bar{Y}}_{p4}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_4^2, \quad R_4 = \frac{\bar{X}C_x}{\bar{X}C_x+D_4} \quad MSE(\hat{\bar{Y}}_{p4}) = \frac{(1-f)}{n}(R_4^2S_x^2+S_y^2(1-\rho^2)).$$

$$B(\hat{\bar{Y}}_{p5}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_5^2, \quad R_5 = \frac{\bar{X}C_x}{\bar{X}C_x+D_5} \quad MSE(\hat{\bar{Y}}_{p5}) = \frac{(1-f)}{n}(R_5^2S_x^2+S_y^2(1-\rho^2)).$$

$$B(\hat{\bar{Y}}_{p6}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_6^2, \quad R_6 = \frac{\bar{X}C_x}{\bar{X}C_x+D_6} \quad MSE(\hat{\bar{Y}}_{p6}) = \frac{(1-f)}{n}(R_6^2S_x^2+S_y^2(1-\rho^2)).$$

$$B(\hat{\bar{Y}}_{p7}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_7^2, \quad R_7 = \frac{\bar{X}C_x}{\bar{X}C_x+D_7} \quad MSE(\hat{\bar{Y}}_{p7}) = \frac{(1-f)}{n}(R_7^2S_x^2+S_y^2(1-\rho^2)).$$

$$B(\hat{\bar{Y}}_{p8}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_8^2, \quad R_8 = \frac{\bar{X}C_x}{\bar{X}C_x+D_8} \quad MSE(\hat{\bar{Y}}_{p8}) = \frac{(1-f)}{n}(R_8^2S_x^2+S_y^2(1-\rho^2)).$$

$$B(\hat{\bar{Y}}_{p9}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_9^2, \quad R_9 = \frac{\bar{X}C_x}{\bar{X}C_x+D_9} \quad MSE(\hat{\bar{Y}}_{p9}) = \frac{(1-f)}{n}(R_9^2S_x^2+S_y^2(1-\rho^2)).$$

$$B(\hat{\bar{Y}}_{p10}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{10}^2, \quad R_{10} = \frac{\bar{X}C_x}{\bar{X}C_x+D_{10}} \quad MSE(\hat{\bar{Y}}_{p10}) = \frac{(1-f)}{n}(R_{10}^2S_x^2+S_y^2(1-\rho^2)).$$

Comparisons with existing ratio estimators

By observing the expressions for the mean square error (MSE) of both the proposed estimators and the existing ones, we have established the conditions under which the efficiency of the proposed estimators is also efficient with the existing modified ratio estimators.

$$MSE(\hat{\bar{Y}}_{pj}) \le MSE(\hat{\bar{Y}}_i),$$

$$\frac{(1-f)}{n}(R_{pj}^2S_x^2+S_y^2(1-\rho^2)) \le \frac{(1-f)}{n}(R_i{}^2S_x^2+S_y^2(1-\rho^2)),$$

$$R_{pj}^2S_x^2 \le R_i^2S_x^2,$$

$$R_{pj} \le R_i,$$

Where

j=1,2,...,10 and i=1,2,....,10

Empirical Study

The illustration has been affianced from the book entitled "Theory and Analysis of Sample Survey Designs" by Singh and Chaudhary (1986) [13].

Where N=34 (n=20) $\bar{Y}$=856.4117 $\bar{X}$= 199.4412

ρ=0.4453 Cx=0.7531

D_1=60.6000 D_2=83.0000 D_3=102.7000 D_4=111.2000 D_5=142.5000 D_6=210.2000 D_7=264.5000 D_8=304.4000 D_9=373.2000 D_{10}=634.0000

Estimators	Constant	Bias	MSE
$\hat{\bar{Y}}_r$	4.294	4.940	10960.76
$\hat{\bar{Y}}_1$	4.294	10.002	17437.65
$\hat{\bar{Y}}_2$	4.278	9.927	17373.31
$\hat{\bar{Y}}_3$	4.285	9.957	17399.52
$\hat{\bar{Y}}_4$	4.281	9.943	17387.08
$\hat{\bar{Y}}_5$	4.258	9.834	17294.19
$\hat{\bar{Y}}_{p1}$	0.712	0.274	9068.58
$\hat{\bar{Y}}_{p2}$	0.644	0.224	9025.63
$\hat{\bar{Y}}_{p3}$	0.594	0.190	8996.90
$\hat{\bar{Y}}_{p4}$	0.574	0.178	8986.46
$\hat{\bar{Y}}_{p5}$	0.513	0.142	8955.55
$\hat{\bar{Y}}_{p6}$	0.417	0.094	8914.08
$\hat{\bar{Y}}_{p7}$	0.362	0.071	8894.42
$\hat{\bar{Y}}_{p8}$	0.330	0.059	8884.24
$\hat{\bar{Y}}_{p9}$	0.287	0.045	8871.83
$\hat{\bar{Y}}_{p10}$	0.191	0.020	8850.71

Conclusion

Based on the empirical study mentioned above, it is evident that our proposed estimators exhibit also an efficiency compared to classical and existing estimators. This is reflected in their lower mean square error (MSE) and bias in comparison to classical and existing estimators. Notably, the efficiency of our estimators is further underscored by the application of the percent relative efficiency (PRE) criterion. The results reveal a consistently higher Percent Relative Efficiency (PRE) for our proposed estimators, establishing their desirability and preference for practical applications. This heightened efficiency implies that our estimators perform competing methods in terms of accuracy and precision, making them reliable tools for statistical inference. The study also emphasizes the positive impact of incorporating auxiliary variable parameters. The inclusion of these variables is identified as a key factor contributing to the observed improvement in estimator performance. This suggests that the utilization of additional information through auxiliary variables enhances the accuracy and robustness of the estimators, thereby providing a more comprehensive and reliable analysis

Application of research: Estimating population mean by using some auxiliary information

Research Category: Agricultural Statistics

Acknowledgement / Funding: Authors are thankful to Division of Agricultural Statistics, Sher-e-Kashmir University of Agricultural Sciences and Technology, Shalimar, Srinagar, 190025, Jammu and Kashmir, India

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Research project name or number: Research station study

Percentage relative efficiency (PRE) of proposed estimators with existing estimators

	$\hat{\bar{Y}}_{p1}$	$\hat{\bar{Y}}_{p2}$	$\hat{\bar{Y}}_{p3}$	$\hat{\bar{Y}}_{p4}$	$\hat{\bar{Y}}_{p5}$	$\hat{\bar{Y}}_{p6}$	$\hat{\bar{Y}}_{p7}$	$\hat{\bar{Y}}_{p8}$	$\hat{\bar{Y}}_{p9}$	$\hat{\bar{Y}}_{p10}$
$\hat{\bar{Y}}_r$	120.8651	121.4403	121.8281	121.9696	122.3907	122.9600	123.2318	123.3731	123.5455	123.8404
$\hat{\bar{Y}}_1$	192.2863	193.2013	193.8183	194.0434	194.7133	195.6190	196.0515	196.2762	196.5506	197.0197
$\hat{\bar{Y}}_2$	191.5768	192.4884	193.1032	193.3275	193.9949	194.8972	195.3281	195.5520	195.8254	196.2928
$\hat{\bar{Y}}_3$	191.8658	192.7788	193.3945	193.6191	194.2875	195.1913	195.6228	195.8470	196.1208	196.5889
$\hat{\bar{Y}}_4$	191.7287	192.6410	193.2562	193.4807	194.1486	195.0517	195.4829	195.7070	195.9806	196.4484
$\hat{\bar{Y}}_5$	190.7044	191.6118	192.2237	192.4470	193.1114	194.0097	194.4386	194.6614	194.9336	195.3988

Author Contributions: All authors equally contributed

Author statement: All authors read, reviewed, agreed and approved the final manuscript. Note-All authors agreed that- Written informed consent was obtained from all participants prior to publish / enrolment

Study area / Sample Collection: Sher-e-Kashmir University of Agricultural Sciences and Technology, Shalimar, Srinagar, 190025, Jammu and Kashmir, India

Cultivar / Variety / Breed name: Nil

Conflict of Interest: None declared

Ethical approval: This article does not contain any studies with human participants or animals performed by any of the authors.
Ethical Committee Approval Number: Nil

References

[1] Cochran W.G. (1940) *Sampling Techniques, 3edn, Wiley Eastern Limited, New York.*
[2] Upadhyaya L.N. & Singh H. (1999) *Biometrical Journal*, 41(5), 627-636.
[3] Singh H.P., Tailor R., Tailor R. & Kakran M. (2004) *Journal of the Indian Society of Agricultural Statistics*, 58(2), 223-230.
[4] Sisodia B.V.S. & Dwivedi V.K. (2012) *Journal of the Indian Society of Agricultural Statistics*, 33(1), 13-18.
[5] Subramani J. & Kumarapandiyan G. (2012a) *International Journal of Probability and Statistics*, 1(4), 111-118.
[6] Maqbool S., Raja T.A. and Javaid S. (2016) *International Journal of Agricultural and Statistical Sci*., 12(1), 95-97.
[7] Subzar M., Bouza C.N., Maqbool S., Raja T.A., Para B.A. (2019) *Revista Investigación Operaci*, 40(2), 201-209
[8] Subzar M., Raja T.A., Maqbool S. and Nazir N. (2016) *Int. J. Agricultural Stat. Sci.,* 12, Sup(1), 221-225.
[9] Subzar M., Abid M., Maqbool S., Raja T.A., Mir S., Lone B.A. (2017) *International Journal of Modern Mathematical Sciences*, 15(2), 187-205.
[10] Raja T.A., Maqbool S. and Subzar M. (2023) *International Journal of Agriculture Sciences*, 15(2), 12199-12203.
[11] Raja T.A., Subzar M., Maqboo S. and Hakak A.A.(2017) *International Journal of Mathematics and Statistics Invention*, 5(1), 58-61.
[12] Kadilar C. & Cingi H. (2004) *Applied Mathematics and Computation*, 151, 893-902.
[13] Kadilar C. & Cingi H. (2006) *Hacettepe Journal of Mathematics and Statistics*, 35(1), 103-109.
[14] Singh D. & Chaudhary F. S. (1986) *Theory and Analysis of Sample Survey Designs, 1edn, New Age International Publisher, India*.