

INDEXING OF ONLINE SIGNATURES

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Abstract- In this paper, a model for representation and indexing of online signatures for person identification is proposed. In some applications, where the database is supposed to be very large, the identification process typically has an unacceptably long response time. A solution to speed up the identification process is to design an indexing model prior to identification which reduces the number of candidate hypotheses to be considered during matching by the identification algorithm. In this paper, we study the suitability of Kd-tree indexing mechanism for person identification based on online signatures. For representation of online signatures, we considered a set of 100 global features of (MCYT online signature database) online signature and index by Kd-tree. Experimental results reveal that indexing prior to identification is faster than conventional identification method in terms of time for online signatures.

Key words - Biometrics, Online Signatures, Indexing, Kd-tree, Person Identification.

INTRODUCTION

Automatic user identity recognition based on biometrics has become a focus of interest both for research community and commercial purposes in recent years. Among the biometrics used, handwritten signature based biometric system is of particular importance as it is the widely accepted method for endorsing financial transactions. An inherent advantage of a signature based biometric system is that the handwritten signature has been established as an acceptable form of personal identification method. It has been used for decades in civilian applications such as banking, business transactions, acknowledgment of goods/services received due to its acceptance in legal and social levels.

Handwritten signature is a behavioral biometric that changes over a period of time and is influenced by the physical and emotional conditions of the signatories. Signatures of some people vary substantially and even successive impressions of their signatures are significantly different. Signatures are composed of special characters and flourishes, therefore most of the time they can be unreadable. Also, intrapersonal variations make it necessary to analyze them as complete images but not as letters and words put together. As signatures are the primary mechanism both for authentication and authorization in legal transactions, the need for research in efficient automated solutions for signature identification and verification has increased in recent years.

There are two main research fields in signature biometric: signature identification and signature verification. The signature identification problem is on identifying the author of a signature. In this problem, entire signature database is searched to establish the identity of a given signer [5]. This task is different from signature verification. The signature verification system validates a person's identity by comparing the captured signature data with

their own signatures stored in the database. Automatic signature verification is an established research field [26], [34]. In comparison, automatic signature identification has received less attention, despite the potential applications that could use the signature as an identification tool [32, 33]. For example, an automated signature identification system could provide a company with a unique technique for validating the identity of each individual accessing to certain security-sensitive facilities [28]. Other potential signature identification applications are in law-enforcement applications, where the identification of perpetrators is a fundamental requirement of the solution, and in the analysis of some historical documents [21].

There are 2 different categories in handwritten signatures: offline signatures and online signatures. An offline signature is nothing but an image of a signature captured by a camera or obtained by scanning a signature on a paper, whereas an online signature is captured using a special digitizing tablet that can record pen positions along with features like azimuth, elevation and pressure. They are intricate variations in signature samples of a person (intra class variations) and also variations in signature samples of different persons (inter class variations). Therefore, signature samples of a person though look similar are not identical. Hence the problem of person identification based on online signature is a challenging task.

With respect to online signatures, the features can be classified into two types: functional features and parametric features. In functional category, the complete signals that is position, velocity, acceleration, force v/s time etc., are represented by mathematical functions whose values directly constitute a feature set. In parametric category, the parameters which are computed from the signals captured by acquisition devices constitute a feature set. From the research community, various features have been recommended for online

signatures. Feature like position, velocity, acceleration, and pressure [2]. Force [9], direction of pen movements [41] and pen inclination [20] considered as functional features.

The most frequently used parameters in online signatures are the average, the root mean square, the maximum, the minimum values which are generally derived from position, displacement, velocity and acceleration signals [12, 24, 25]. Total signature time duration, pen down time ratio, number of pen ups/pen downs, number of pen lifts, positive/negative time duration of position and X-Y correlation of position, displacement, speed and acceleration [24, 27, 31] are also widely used. Apart from these features, direction based features [11, 24, 31, 39], curvature based features [22, 38], symbolic features [17], moment based features [24, 36], Wavelet Transform [4], Fourier Transform [10, 40] were also used to extract the features. In general, function based systems show better performance than the parameter-based systems but require time consuming matching/comparison procedures. However, the work [2] shows that the parametric approaches are equally competitive when compared to function-based approaches.

However the extracted features should be properly represented in a suitable feature space. The proper representation concerned with deciding the important features and the number of such important features and deciding good data structures to enable efficient/effective storage for subsequent identification purposes. Unfortunately, most of the explored identification models focused on only the accuracy of recognition that too with small databases, neglecting the problems of scalability and speed despite the fact that these are required for large databases. Indeed response time, search and retrieval efficiency in addition to accuracy are also of great importance while deploying an identification model for real time applications. A solution to speed up the identification process is to reduce the number of comparisons required or to design a model which works with constant recognition time regardless of the size of the database. To reduce the number of comparisons, certain classification or indexing techniques are required. A common technique to achieve this is to partition the whole database into a number of bins based on some predefined classes [29]. However, in the case of a real-time situation, where the samples are added frequently into the database, the binning approach poses a potent problem of having to repartition the whole database into bins and moreover as the database size increases the repartitioning time grows quickly [29]. Hence, intrinsic information in the biometric data can be used for efficient and effective indexing and only a few researchers studied the use of indexing to index the entire biometric data in such a way that matching phase deals with only a small subset of the entire database [14, 18, 30]. Basically, signature identification system can be optimized when the query signature is matched with only few potential best hypotheses instead of matching with all the signatures in the database. Thus an efficient and effective indexing mechanism for online signatures is still a challenging work in the situations of a large signature database.

So far, Han and Sethi worked on offline signature retrieval [19]. They work on handwritten signatures and use a set of geometrical and topological features to map a signature onto 2D-strings [8]. However, 2D-strings are not invariant to similarity transformations and any retrieval systems based on them are hindered by many bottlenecks [18]. There are several approaches for perceiving spatial relationships such as nine directional lower triangular matrix (9DLT) [7] and triangular spatial relationship (TSR) [13] etc. In order to overcome the said problem, Guru et. al., proposed an online signature retrieval model using global features based on SIM_R [16]. Prakash and Guru proposed offline signature retrieval model based on spatial topology of geometric centers, which quickly retrieve the signatures from the database for a given query in the decreasing order of their spatial similarity with the query [35]. Jayaraman et al., proposed indexing scheme for signature with other modalities using feature level fusion. But the indexing performance is not stated clearly in that work [23]. In our previous work, we have proposed indexing mechanism for offline signatures [15]. To the best of our knowledge, no work on indexing of online signature is proposed. Hence, in this work we focus on designing an indexing mechanism for online signatures for optimizing subsequent robust recognition system.

The organization of the paper is as follows. In section 2, proposed indexing model based on Kd-tree for MCYT online signatures is presented. Experimental results with time analysis are given in section 3. Finally the summary of the proposed work with future avenues are provided in section 4.

PROPOSED MODEL

In this paper, we study the set of 100 global features (MCYT online signature database) of online signature for person identification. Initially, we computed the identification accuracy of online signatures of a person using the set of 100 global features. Later we study the effect of Kd-tree based indexing approach for online signatures useful for person identification.

Different approaches are considered in the literature in order to extract signature information; they can be divided into [34]: *i*) Function-based approaches, in which signal processing methodology is applied to the dynamically acquired time sequences (velocity, acceleration, force or pressure), and *ii*) Feature based or parameter based approaches, in which statistical parameters are derived from the acquired information; regarding these, one can specify also different levels of classification, so it is possible to use and combine shape-based global static (aspect ratio, center of mass or horizontal span ratio), global dynamic (total signature time, time down ratio or average speed) or local (stroke direction, curvature or slope tangent) parameters. A sample online signature of the MCYT signature subcorpus along with the captured values of x, y, pressure, azimuth and elevation are shown in Fig. (1). We have considered a set of 100 global features of online signatures for the experimentation purpose and is shown in Table 1. For more details of the online signature features are referred to [1, 3, 27].

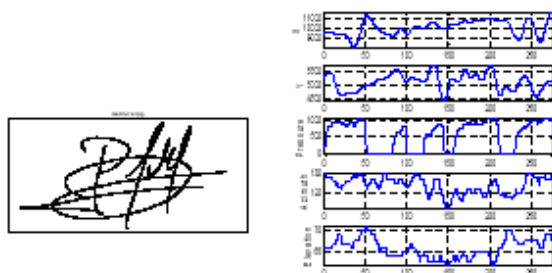


Fig.1- A sample online signature from MCYT signature corpus.

Once we get the feature set, we store it in the database. However storing in the database in an efficient manner is required such that the possible candidate list is selected for the matching process should be very much less. Hence, there is a need of backend tool called indexing mechanism which stores the data in some predefined manner so that during matching phase only a few potential candidates are selected. Hence, in this paper we study the suitability of Kd-tree based approach for indexing the obtained features. The following subsection provides an overview of Kd-tree indexing method.

Kd-tree (k-dimensional tree) is a space-partitioning data structure for organizing points in a k-dimensional space. It is a useful data structure for several applications, involving a multidimensional search key. Kd-trees are a special case of binary space partitioning (BSP) trees and it uses only splitting planes that are perpendicular to one of the coordinate system axes. In BSP trees, arbitrary splitting planes can be used. In the typical definition, every node of a Kd-tree, from the root to the leaves, stores a point, but in BSP trees, leaves are typically the only nodes that contain points (or other geometric primitives). As a consequence, each splitting plane must go through one of the points. These are a variant that store data only in leaf nodes [37]. Kd-tree for a set of 'n' feature points uses $O(n)$ storage and can be constructed in $O(n \log n)$.

In this proposed method, multi-dimensional feature vector is obtained from online signatures and is indexed using the Kd-tree. Kd-tree is an appropriate data structure for biometric identification system particularly in the analysis of execution of range search algorithm and it decreases the search time as it is supporting the range search with a good pruning. When query feature vector of multi-dimension is given, range search is invoked using Kd-tree to retrieve top matches that lie within distance 'd' from the query. These top matches are subsequently used for signature identification. Fig. (2) shows the block diagram of proposed Kd-tree based indexing mechanism of online signatures for person identification.

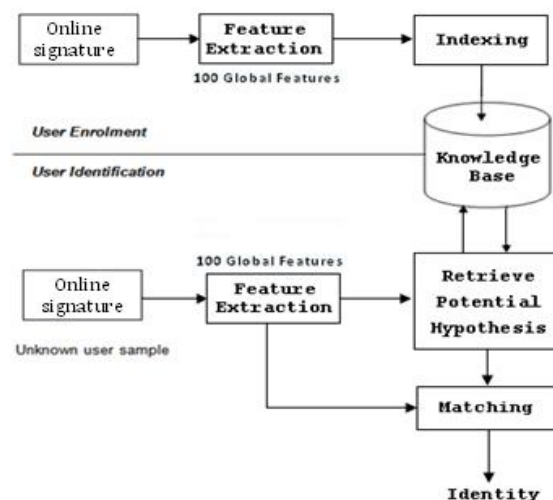


Fig. 2- Block diagram of indexing model for online signatures.

EXPERIMENTAL RESULTS

The proposed indexing approach based on Kd-tree is tested on MCYT online signature database. The MCYT signature subcorpus consists of online signature samples of 330 individuals. For each individual there are totally 50 signatures, out of which 25 are genuine and 25 are skilled forgeries. For experimental purpose, we have considered only genuine samples and thus totally it forms a signature dataset of 8250 (330 x 25) online signatures.

Feature vectors of 100 dimensions of online signatures are obtained and indexed through Kd-tree. The output of an indexing algorithm is the set of top hypotheses [6]. If the corresponding signature is in the list of top hypotheses, we take the indexing result as a correct result. Hence, the measures such as False Acceptance Rate (FAR) and False Rejection Rate (FRR) which are generally used for verification are not suitable for evaluating the results of an indexing algorithm [6].

In this paper, we use Correct Index Power (CIP) as the performance evaluation measure for indexing. CIP is the ratio of the correct queries to all queries. The system is trained using 40%, 60% and 80% samples per user and is tested with remaining 60%, 40% and 20% samples per user respectively. When a query feature vector 'Q' of n (n = 100) dimensions is given, it retrieves top matches that lie within distance 'd' from query 'Q' and top matches are subsequently used for signature identification. Earlier to this, the identification accuracy for different training sets using conventional method is computed and the obtained results are tabulated in Table 2. The graphs of CIP v/s percentage of database search of online signatures for 40%, 60% and 80% training samples are shown in Fig. (3). From Fig. (3), we can infer that good CIP can be achieved by indexing set of 100 features for the purpose of signature identification.

Table – 2 - Identification accuracy of online signatures for different training sets.

Training	Identification Accuracy
40%	72.59 %
60%	78.03 %
80%	81.58 %

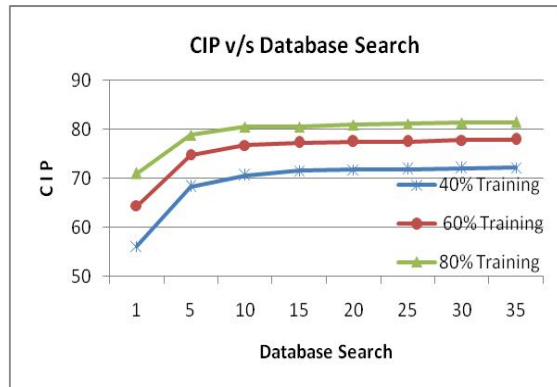


Fig. 3- Indexing performance of MCYT based online signatures using the set of 100 features for different training sets.

The beauty of our indexing scheme lies in its efficiency from the point of search time. The time analysis for indexing based and conventional identification methods for 80% training is tabulated in Table 3. From Table 3, it is clear that, the proposed indexing method reduces the search time, which supports range search with a good pruning. From Table 3, it is clear that, the test sample is searched within a database using negligible amount of time when compared to conventional signature identification with same accuracy. Hence we claim that indexing prior to online signature identification is faster than conventional identification of online signature. The percentage of time reduction for identification of online signatures using the Kd-tree indexing model against the conventional identification method for different training sets is shown in Table 4.

Table – 3- Time analysis of methods with proposed indexing and without indexing.

	Percentage of Database	Time in Seconds	Accuracy in %
With Indexing	1 %	0.0027	71.03
	5 %	0.0037	78.79
	10 %	0.0045	80.42
	15 %	0.0056	80.85
	20 %	0.0063	80.91
	25 %	0.0070	81.09
	30 %	0.0083	81.27
	35 %	0.0092	81.39
	83.62 %	0.0343	81.58
Conventional identification with 100% scanning		0.9251	81.58

Table – 4 - The percentage of time reduction for identification of online signatures using the proposed indexing model against the conventional identification method.

Training	Identification time in seconds		Time reduction in %
	Conventional	Indexing	
40%	0.5213	0.0211	95.95
60%	0.7326	0.0235	96.79
80%	0.9251	0.0343	96.29

CONCLUSIONS

In this paper, we proposed Kd-tree based indexing approach to index the online signatures. Experiments are conducted on MCYT online signature database and the obtained experimental results are more encouraging. Searching and insertion of new nodes in Kd-tree is straightforward. Deletion may cause re-organization of the tree under the deleted node, thus it can be more complicated. The Kd-tree structure depends heavily on the insertion order of the feature points. As the division of hyperplanes is defined by the position of the points, dividing the plane may not be at the best possible positions, resulting in an unbalanced tree. On the other hand, the extracted features are of high dimension, hence we are currently studying the effect of dimensionality reduction. Along with that, feature selection is also in progress. In addition to this, our future work is to incorporate the other balanced multidimensional data structures to index online signatures.

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Table – 1- The set of 100 global features of MCYT online signatures.

Ranking	Feature Description	Ranking	Feature Description
1	signature total duration T_s	2	$N(\text{pen-ups})$
3	$N(\text{sign changes of } dx/dt \text{ and } dy/dt)$	4	average jerk \bar{J}
5	standard deviation of a_y	6	standard deviation of v_y
7	(standard deviation of y)/ Δ_y	8	$N(\text{local maxima in } x)$
9	standard deviation of a_x	10	standard deviation of v_x
11	j_{rms}	12	$N(\text{local maxima in } y)$
13	$t(2^{\text{nd}} \text{ pen-down})/T_s$	14	(average velocity \bar{v})/ $v_{x,\text{max}}$
15	$\frac{A_{\min} = (y_{\max} - y_{\min})(x_{\max} - x_{\min})}{\left(\Delta_x = \sum_{i=1}^{\text{pendowns}} (x_{\max i} - x_{\min i})\right)\Delta_y}$	16	$(x_{\text{last pen-up}} - x_{\min})/\Delta_x$
17	$(x_{1^{\text{st}} \text{ pen-down}} - x_{\min})/\Delta_x$	18	$(y_{\text{last pen-up}} - y_{\min})/\Delta_y$
19	$(y_{1^{\text{st}} \text{ pen-down}} - y_{\min})/\Delta_y$	20	$(T_w \bar{v})/(y_{\max} - y_{\min})$
21	$(T_w \bar{v})/(x_{\max} - x_{\min})$	22	(pen-down duration T_w)/ T_s
23	$v/v_{y,\text{max}}$	24	$(y_{\text{last pen-up}} - y_{\max})/\Delta_y$
25	$T((dy/dt)/(dx/dt) > 0) / T((dy/dt)/(dx/dt) < 0)$	26	\bar{v} / v_{\max}
27	$(y_{1^{\text{st}} \text{ pen-down}} - y_{\max})/\Delta_y$	28	$(x_{\text{last pen-up}} - x_{\min})/\Delta_x$
29	(velocity rms v)/ v_{\max}	30	$(x_{\max} - x_{\min})\Delta_y / (y_{\max} - y_{\min})\Delta_x$
31	(velocity correlation $v_{x,y}$)/ v_{\max}^2	32	$T(v_y > 0/\text{pen-up})/T_w$
33	$N(v_x = 0)$	34	direction histogram s_1
35	$(y_{2^{\text{nd}} \text{ local max}} - y_{1^{\text{st}} \text{ pen-down}})/\Delta_y$	36	$(x_{\max} - x_{\min})/x_{\text{acquisition range}}$
37	$(x_{1^{\text{st}} \text{ pen-down}} - x_{\max})/\Delta_x$	38	$T(\text{curvature} > \text{Threshold}_{\text{curv}})/T_w$
39	(integrated abs. centr. acc. a_c)/ a_{\max}	40	$T(v_x > 0)/T_w$
41	$T(v_x < 0/\text{pen-up})/T_w$	42	$T(v_x > 0/\text{pen-up})/T_w$
43	$(x_{3^{\text{rd}} \text{ local max}} - x_{1^{\text{st}} \text{ pen-down}})/\Delta_x$	44	$N(v_y = 0)$
45	(acceleration rms a)/ a_{\max}	46	(standard deviation of x)/ Δ_x
47	$T((dx/dt)(dy/dt) > 0) / T((dx/dt)(dy/dt) < 0)$	48	(tangential acceleration rms a_t)/ a_{\max}
49	$(x_{2^{\text{nd}} \text{ local max}} - x_{1^{\text{st}} \text{ pen-down}})/\Delta_x$	50	$T(v_y < 0/\text{pen-up})/T_w$
51	direction histogram s_2	52	$t(3^{\text{rd}} \text{ pen-down})/T_s$
53	(max distance between points)/ A_{\min}	54	$(y_{3^{\text{rd}} \text{ local max}} - y_{1^{\text{st}} \text{ pen-down}})/\Delta_y$
55	$(\bar{x} - x_{\min})/x$	56	direction histogram s_5
57	direction histogram s_3	58	$T(v_x < 0)/T_w$
59	$T(v_y > 0)/T_w$	60	$T(v_y < 0)/T_w$
61	direction histogram s_8	62	$(1^{\text{st}} t(v_{x,\text{min}}))/T_w$
63	direction histogram s_6	64	$T(1^{\text{st}} \text{ pen-up})/T_w$
65	spatial histogram t_4	66	direction histogram s_4
67	$(y_{\max} - y_{\min})/y_{\text{acquisition range}}$	68	$(1^{\text{st}} t(v_{x,\text{max}}))/T_w$
69	(centripetal acceleration rms a_c)/ a_{\max}	70	spatial histogram t_1
71	$\theta(1^{\text{st}} \text{ to } 2^{\text{nd}} \text{ pen-down})$	72	$\theta(1^{\text{st}} \text{ pen-down to } 2^{\text{nd}} \text{ pen-up})$
73	direction histogram s_7	74	$t(j_{x,\text{max}})/T_w$
75	spatial histogram t_2	76	$j_{x,\text{max}}$
77	$\theta(1^{\text{st}} \text{ pen-down to last pen-up})$	78	$\theta(1^{\text{st}} \text{ pen-down to } 1^{\text{st}} \text{ pen-up})$
79	$(1^{\text{st}} t(x_{\max}))/T_w$	80	\bar{J}_x
81	$T(2^{\text{nd}} \text{ pen-up})/T_w$	82	$(1^{\text{st}} t(v_{\max}))/T_w$
83	$j_{y,\text{max}}$	84	$\theta(2^{\text{nd}} \text{ pen-down to } 2^{\text{nd}} \text{ pen-up})$
85	j_{max}	86	spatial histogram t_3
87	$(1^{\text{st}} t(v_{y,\text{min}}))/T_w$	88	$(2^{\text{nd}} t(x_{\max}))/T_w$
89	$(3^{\text{rd}} t(x_{\max}))/T_w$	90	$(1^{\text{st}} t(v_{y,\text{max}}))/T_w$
91	$t(j_{\text{max}})/T_w$	92	$t(j_{y,\text{max}})/T_w$
93	direction change histogram c_2	94	$(3^{\text{rd}} t(y_{\max}))/T_w$
95	direction change histogram c_4	96	\bar{J}_y
97	direction change histogram c_3	98	$\theta(\text{initial direction})$
99	$\theta(\text{before last pen-up})$	100	$(2^{\text{nd}} t(y_{\max}))/T_w$