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MAGNETOHYDRODYNAMIC BLOOD FLOW IN A NARROW TUBE

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Abstract- The present paper is devoted to study the flow of an incompressible, viscous, electrically conducting fluid in the presence of transverse magnetic field. The governing equations of motion in terms of cylindrical polar coordinates are reduced to an ordinary differential equation by using dimensionless parameters and then solved analytically. Exact solutions for the axial velocity, flow rate and dynamic viscosity are presented. Graphical representations of the results in terms of dimensionless parameters are outlined.
The main result of this work is that, the effect of the magnetic field is to decrease the velocity profile and flow rate. Therefore, the velocity profile becomes more parabolic. The blood flow in the presence of a transverse magnetic field is taken as an example of this type of flow.
Magnetohydrodynamics (blood) flow in a narrow tube is described using two layered (two-phase) model. The model consists of a central core region enriched with various types of blood cells such as red blood cells (RBCs) and a cell free layer (plasma) surrounding the core.
The bluntness appears in velocity profile and this bluntness decrease by increasing magnetic field.
Keywords- Confusion matrix, Data Mining, Decision tree, Neural Network, stacking ensemble, voted perceptron

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Introduction

The study of the MHD flow problems through a tube has found an applications in many fields like MHD power generation, blood flow measurements, etc. In our problem we have obtained the exact solution of viscous incompressible flow through a tube and the magnetic field is transverse. The major part of the thesis deals with the analytical solution of the blood flow in the tube under a transverse magnetic field.

The theoretical studies of blood flow in a tube (a narrow vessel) phenomena are very useful for the diagnosis of a number of cardiovascular diseases in human physiology and for other clinical purposes. It is well known that blood behaves differently, in which Newtonian behavior is expected and in small vessels where non-Newtonian effect. Blood can be regarded as a magnetic fluid appeared in which red blood cells, (RBCs), magnetic in nature due to iron oxide in it. So when we apply a magnetic field on blood the only components of blood response to these field are RBCs, and this occur in-vitro (the blood represented as a suspension fluid). The movement of blood in an externally applied magnetic field is governed by the low of magneto hydrodynamics (MHD). When the body is subjected to a magnetic field the charged particles of the blood flowing transversally to the field get deflected by the Lorenz force, thus including electrical currents and voltages across the vessel walls. Therefore the interactions between these induced currents and applied magnetic field can cause a reduction of blood flow rate.

Magnetic field interactions with blood flow have been demonstrated by many authors. Korchevskii and Marochnik [1] first proposed a velocity profile solution for blood flow as a Newtonian fluid between two parallel plates under a constant pressure gradient with a perpendicular magnetic field. Vardanyan [2] derived an approximation steady solution where velocity profile and flow rate were calculated by neglecting the induced fields. More recent studies were based on these founding works such as Keltner et al. [3].

When blood flows through tubes, the two-phase nature of blood as a suspension becomes important as the diameter of the RBCs becomes comparable to the tube diameter. The following are some of the effects observed in vitro and in vivo: (i) Fahraeus-

Lindqvist effect: dependence of apparent viscosity on tube diameter; (ii) Fahraeus effect: dependence of tube or vessel hematocrit on tube diameter; (iii) Existence of a cell-free layer near the wall: (iv) Blunt velocity profile. Nair et al. [4] used a two-phase model for the blood in modeling transport of oxygen in the arterioles. They considered a cell-rich core surrounded by a cell-free plasma layer. In the cell-rich core, the radial hematocrit distribution was expressed as a power law profile with maximum at the center of the tube. A parabolic velocity profile was taken for the plasma in the cell-free layer. Different velocities were used for the plasma and RBCs in the cell-rich core. The parabolic profiles were slightly blunted.

Seshadri and Jaffrin [5] modeled the outer layer as cell-depleted, having a lower hematocrit than in the core. The apparent viscosity and the mean tube hematocrit were taken from the measurements obtained in glass tubes. The concentration of RBCs in the cell-depleted layer was assumed to be 50% of that in the core. Gupta et al. [6] divided the outer layer into a cell-free plasma layer and cell-depleted layer. In both these studies, the velocity profile in the core was assumed to follow as a power law.

Most mathematical descriptions of a steady blood flow in a cylindrical tube using a two-phase fluid model. The gradient in a shear rate create a force effect on cells. so, the RBCs flow toward the axis of high velocity (axial migration) and create a cell-free layer near from the wall of tube. Axial migration depends on particle size, velocity gradient, and the radius of the tube. Therefore, the viscosity in the core with RBCs is larger than that in the free layer with plasma because the particle size is large in core. The axial migration leads to bluntness in velocity profile and this different from parabolic velocity profile in a homogenous fluid.

Formulation of the Problem

Consider the motion of an electrically conducting viscous fluid in a circular tube of radius R and length L in the presence of a transverse magnetic field, see. Fig. 1-

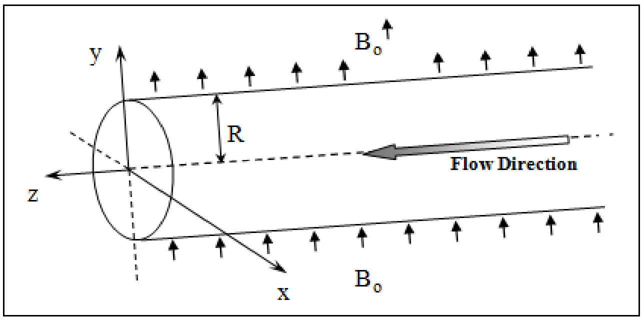

Fig. 1- Flow Model Geometry

Dynamical Equation of Motion

The steady flow of viscous fluid in the presence of a magnetic field is governed by the conservation laws of mass and momentum which are:

$$\nabla.\underline{v} = 0 \tag{1}$$

$$\rho(\underline{v}.\nabla\underline{v}) = -\nabla p + \mu\nabla^2\underline{v} + \underline{f}, \tag{2}$$

where $\underline{v}$ is the velocity vector, $\underline{f}$ is the body force per unit volume; ρ is the density, and ∇p is the pressure gradient. Therefore, f includes any external forces such as gravity or electromagnetic forces. In our problem, since the fluid is electrically conducting and there is an applied magnetic field, the most common type of body force which acts on a fluid is due to electromagnetic effects. Therefore, the presence of the magnetic field requires that an additional force must be included in equation (2) aside from the usual pressure and shear forces. The added force, is

$$\underline{f} = (\rho_c\underline{E} + \underline{J}\times\underline{B}), \tag{3}$$

where ρ_c is the charge density and $\underline{B}$ is the total magnetic field vector. The current density $\underline{J}$ may be expressed by the generalized ohm's law, i.e.

$$\underline{J} = \sigma(\underline{E} + \underline{v}\times\underline{B}), \tag{4}$$

where σ is the electrical conductivity of the fluid. The terms σ $\underline{E}$ and σ ($\underline{v}\times\underline{B}$), represent the conduction and induction current. For simplicity, we assume that the wall of tube is an electrically insulator. Next, the externally applied electric field is arranged to be zero. Finally, it is postulated that, the induced fields are negligibly small. Therefore; Eq. (3) and Eq. (4) take the form.

$$\underline{f} = \underline{J}\times\underline{B}, \tag{5}$$

$$\underline{J} = \sigma(\underline{v}\times\underline{B}), \tag{6}$$

the equations describing the steady motion of the fluid under consideration, Eq. (2), are being

$$\rho(\underline{v}.\nabla\underline{v}) = -\nabla p + \mu\nabla^2\underline{v} + \underline{J}\times\underline{B}, \tag{7}$$

The term $\underline{v}.\nabla\underline{v}$ vanishes in equation (2), also the pressure p is a function in z position only. In view of these assumptions, Eq. (7), takes the form.

$$-\nabla p + \mu\nabla^2\underline{v} + \sigma(\underline{v}\times\underline{B})\times\underline{B} = 0, \tag{8}$$

Let us consider a cylindrical coordinate system (r, θ, z) having the z- axis coincident with the tube axis. The velocity components can be written as $\underline{V}$ = [0, 0, w(r)] which satisfies the equation of continuity. The external constant magnetic field is applied transversally such as $\underline{B} = (B_o\cos\theta, -B_o\sin\theta, 0)$ and the wall of tube is considered to be insulator. The operator ∇^2 in cylindrical coordinate take the form:

$$\nabla^2 = \frac{1}{r}\left\{\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial}{\partial\theta}\left(\frac{1}{r}\frac{\partial}{\partial\theta}\right) + \frac{\partial}{\partial z}\left(r\frac{\partial}{\partial z}\right)\right\},$$

So

$$(\nabla^2\underline{v})_z = \frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right),$$

Taking into account these assumptions the velocity profile can be defined by the z- component of the momentum equation, Eq. (8)

as:

$$-\frac{dp}{dz}+\mu\left(\frac{d^2w}{dr^2}+\frac{1}{r}\frac{dw}{dr}\right)-\sigma B_o{}^2 w=0$$

$$0\le r\le R \quad , \qquad (9)$$

Boundary Conditions

Equation (9), give a complete description of the physical occurrence within the fluid. But to complete the statement of the problem, it still remains to specify the boundary conditions. For the present case the velocity is maximum in the center of the tube and decrease when approach to the wall. These conditions are given by:

$$w=0 \quad , \quad \text{at} \quad r=R \quad , \qquad (10)$$

$$\frac{dw}{dr}=0 \quad , \quad \text{at} \quad r=0 \quad , \qquad (11)$$

Equation (11) shows that, the velocity w is bounded on the z- axis and because of symmetry the velocity gradient vanishes along the axis of the tube.

Dimensionless Parameters

The governing equations, Eqs. (9) to (11) to dimensionless form in order to be simple in mathematical handing,

$$\eta=\frac{r}{R} \quad , \qquad f=\frac{\mu L}{\nabla p R^2}w \quad . \qquad (12)$$

In terms of the new variables given in Eq (12) the governing equations, Eq. (9), becomes:

$$\left(\frac{d^2f}{d\eta^2}+\frac{1}{\eta}\frac{df}{d\eta}\right)-m\varphi=\frac{L}{\Delta p}\frac{dp}{dz} \quad , \qquad (13)$$

Let k= $\frac{L}{\Delta p}\frac{dp}{dz}$, $\left(\frac{d^2f}{d\eta^2}+\frac{1}{\eta}\frac{df}{d\eta}\right)-mf=k$, (14)

in Eq. (14), the symbol m represents the magnetic parameter (staurt number) defined by:

$$m=\frac{\sigma R^2 B_o{}^2}{\mu} \quad , \qquad (15)$$

the magnitude of m is an index to the relative importance of magnetic forces. When m = 0, magnetic forces are absent; when m increases, the magnetic forces become increasing.

The boundary conditions imposed on the functions f in terms of the new variables are:

$$f=0 \quad , \quad \text{at} \quad \eta=1 \quad , \qquad (16)$$

$$\frac{df}{d\eta}=0 \quad , \quad \text{at} \quad \eta=0 \quad , \qquad (17)$$

In the state of blood:

Formulation of the Problem

Consider the motion of blood is an incompresspole, an electrically conducting viscous fluid in a circular tube of radius R and length L in the presence of a transverse magnetic field. Blood is represented by a two-phase model consisting of a cylindrical core region of radius r_i and a cell-free layer outside the core containing plasma with an effective viscosity μ_0. The core region contains a suspension of red blood cells in plasma with an effective viscosity μ_i ($\mu_i > \mu_0$), see Fig. 2.

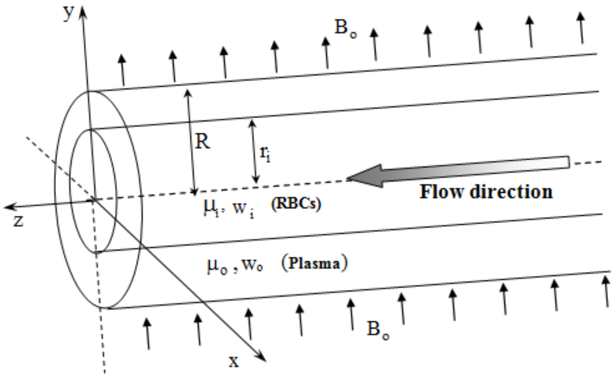

Fig. 2- Flow Model Geometry

Dynamical Equation of Motion

The steady flow of blood as a Newtonian incompressible fluid in the presence of a magnetic field is governed by the conservation laws of mass and momentum in Eq. (1)-(9). When the blood flow through a tube Eq. (9) takes the form:

$$-\frac{dp}{dz}+\mu_i\left(\frac{d^2w_i}{dr^2}+\frac{1}{r}\frac{dw_i}{r}\right)-\sigma B_0{}^2 w_i=0$$

$$0\le r\le r_i \quad , \qquad (18)$$

for the core layer with RBCs and

$$-\frac{dp}{dz}+\mu_0\left(\frac{d^2w_0}{dr^2}+\frac{1}{r}\frac{dw_0}{r}\right)=0$$

$$r_i\le r\le R \quad , \qquad (19)$$

for the cell free (plasma) layer

Boundary Conditions

Equation (18) and (19), give a complete description of the physical occurrence within the fluid. But to complete the statement of the problem, it still remains to specify the boundary conditions. For the present case the boundary conditions are the standard no-slip conditions of velocities and these conditions are given by:

$$w_0=0 \quad , \quad \text{at} \quad r=R \quad , \qquad (20)$$

$$\frac{dw_i}{dr}=0 \quad , \quad \text{at} \quad r=0 \quad , \qquad (21)$$

Equation (21) shows that, the core velocity w_i is bounded on the z- axis and because of symmetry the velocity gradient vanishes along the axis of the tube.

Dimensionless Parameters

The governing equations, Eqs. (18) to (21) to dimensionless form in order to be simple in mathematical handing

$$\eta=\frac{r}{R} \quad , \qquad \alpha=\frac{r_i}{R} \quad . \qquad (22)$$

In which α represents the dimensionless core radius. When a =0, the core layer is absent and the blood is a homogenous Newtonian fluid. When α increase a® 1 the core layer becomes increasingly important which equivalent to blood flow in a large tube. In terms of the new variables given in Eq (22) the governing equations, Eqs. (18) and (19), becomes:

$$\frac{R^2}{\mu_i}\left(-\frac{dp}{dz}\right)+\left(\frac{d^2w_i}{d\eta^2}+\frac{1}{\eta}\frac{dw_i}{d\eta}\right)-mw_i=0 \quad 0\le\eta\le\alpha \qquad , \qquad (23)$$

for the core region and

$$\frac{R^2}{\mu_0}\left(-\frac{dp}{dz}\right)+\left(\frac{d^2w_0}{d\eta^2}+\frac{1}{\eta}\frac{dw_0}{d\eta}\right)=0 \quad \alpha\le\eta\le1 \qquad , \qquad (24)$$

for the plasma region

The boundary conditions imposed on the functions w_i and w_0 in terms of the new variables are:

$$w_0=0, \quad \text{at} \quad \eta=1, \qquad (25)$$

$$\frac{dw_i}{d\eta}=0, \quad \text{at} \quad \eta=0, \qquad (26)$$

Solution of the Problem

Velocity Function

The dimensionless velocity function $f(\eta)$ is obtained from the solution of Eq. (14) subject to the boundary conditions, (16) and (17). This solution takes the form:

$$f(\eta)=\frac{k}{\sqrt{m}}\left[1-\frac{I_0(\sqrt{m}\eta)}{I_0(\sqrt{m})}\right], \quad -1\le\eta\le1 \qquad (27)$$

Flow Rate

The flow rate takes the following form:

$$Q=2\pi\int_0^R w(r)r\,dr \qquad , \qquad (28)$$

with the help of Eq. (27), the rate of flow in dimensionless form is:

$$Q=\frac{2\pi R^4}{\mu}\left(\frac{\Delta p}{L}\right)\int_0^1 f(\eta)\eta\,d\eta, \qquad (29)$$

Since $\int I_0(x)dx=I_1(x),$ so

$$Q=\frac{2\pi R^4}{\mu}\left(\frac{\Delta p}{L}\right)\frac{k}{\sqrt{m}}\left[1-\frac{I_1\sqrt{m}}{I_0\sqrt{m}}\right], \qquad (30)$$

Asymptotic Solutions

In order to discuss the results of the theoretical proposed in this study an asymptotic solutions are used to evaluate the analytical results for velocity profiles and for flow rate obtained in Eqs.(27)-(30) for various values of the parameters involved.

Velocity Function

By using the standard approximation formula of modified Bessel function of zero order:

$$I_0(x)=\sum_{n=0}^{\infty}\frac{1}{n^2!}\left(\tfrac{1}{2}x\right)^{2n} \qquad , \qquad (31)$$

the velocity function can be expressed in the following parabolic distribution:

$$f(\eta)=\frac{k\sqrt{m}}{(4+m)}(1-\eta^2) \qquad , \quad -1\le\eta\le1 \qquad (32)$$

Flow Rate

Since

$$I_1(x)=\sum_{n=0}^{\infty}\frac{1}{n!(n+1)!}\left(\tfrac{1}{2}x\right)^{2n+1} \qquad ,$$

it follows that the flow rate, Eq. (30), approaches the following form:

$$Q=\frac{2\pi R^4 k}{\mu\sqrt{m}}\left(\frac{\Delta p}{L}\right)\left[1-\frac{m\sqrt{m}+8\sqrt{m}}{4(4+m)}\right], \qquad (33)$$

Or

$$Q=\frac{4\pi R^4\Delta pk}{8\mu L}\left[\frac{1}{\sqrt{m}}-\frac{1}{4}+\frac{1}{4+m}\right]. \qquad (34)$$

Apparent Magnetic Viscosity of Viscous fluids

The standard flow rate of viscous fluid in a tube is given by the equation :

$$Q=\frac{\pi R^4}{8\mu}\frac{1}{L}\left[\frac{-dp}{dz}\right] \qquad , \qquad (35)$$

As a result of our study, the rate of flow of magnetic fluid in a tube can be written as:

$$Q=\frac{\pi R^4}{8\mu_{app}L}\left[\frac{-dp}{dz}\right] \qquad , \qquad (36)$$

where μ_{app} is the apparent viscosity of the magnetic fluid which is a measure of viscosity when the magnetic field is applied. The comparison between Eq. (34) and Eq. (36) gives:

$$\mu_{app}=4k\left[\frac{1}{\sqrt{m}}-\frac{1}{4}+\frac{1}{4+m}\right]. \qquad (37)$$

Solution of MHD Blood Problem

Velocity Function

The velocity functions $w_i(\eta)$, $w_0(\eta)$ are obtained from the solution

$$w_i=\frac{I_0\sqrt{m}\eta}{I_0\sqrt{m}\alpha}\left[\frac{R^2}{4\mu_0}\left(\frac{dp}{dz}\right)(1-\alpha^2)+\frac{R^2}{m\mu_i}\left(\frac{dp}{dz}\right)\right]-\frac{R^2}{m\mu_i}\left(\frac{dp}{dz}\right) \qquad .(38)$$

$0 \le \eta \le \alpha$,

solution of the outer region (cell free layer) is:

$$w_0 = -\frac{R^2}{4\mu_0}\left(\frac{dp}{dz}\right)(1-\eta^2) \quad , \quad \alpha \le \eta \le 1 \qquad (39)$$

Asymptotic Solutions

In order to discuss the results of the theoretical model proposed in the study, an asymptotic solution are used to evaluate the analytical results for velocity profiles in Eq. (38)- (39) for various values of the parameters involved.

Velocity Function

By using the standard approximation formula of modified Bessel function of zero order given in Eq.(31):
the velocity function in the core can be expressed in the following parabolic distribution:

$$w_i = w_{max}(1-\beta\eta^2) \quad , \quad \alpha \le \eta \le 1 \quad , \qquad (40)$$

Where

$$w_{max} = \frac{R^2(-dp/dz)}{\mu_0(4+m\alpha^2)}\left[1-\alpha^2\left(1-\frac{\mu_0}{\mu_i}\right)\right],$$

$$\beta = \frac{4\,\mu_0/\mu_i - m(1-\alpha^2)}{4\left[1-\alpha^2\left(1-\frac{\mu_0}{\mu_i}\right)\right]}. \qquad (41)$$

The parameter β represents the deviation from parabolic flow, when m = 0,
β = 1 the velocity profile becomes parabolic throughout the entire cross section of the tube and the fluid become homogenous.

Results and Discussion

Because the goal of this study is to investigate the effect of an external magnetic field on the viscous flow in a tube, we have chosen parameters of the flow from the study of Sharan et al [14]. The radius R of the tube is chosen as R =70 μm.Values of the magnetic parameter m are chosen between 1 and 4.

Axial Velocity Distribution $f(\eta)$

Axial velocity profiles f computed from the present study, eq. (32) is represented graphically in Fig.3. The effect of the magnetic field parameter m is also shown in that Fig. In these Fig. we observe that the velocity profile of viscous fluid is more parabolic and these parabolic decrease by increasing magnetic field parameter m.

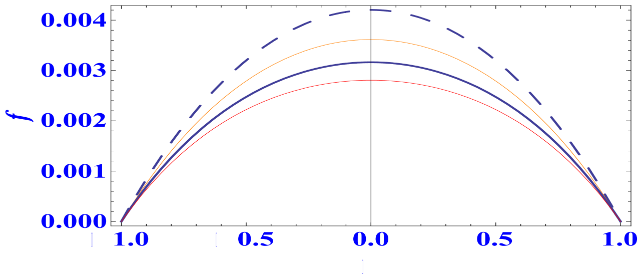

Fig. 3- Velocity profiles at k=0.02, and magnetic parameter (m=1, 2, 3, 4) top to bottom.

Flow Rate Distribution Q (m)

Values of the volumetric flow rate Q are calculated using Eq. (34). The variation of Q against m is shown in Fig. 4 for different values of m taking k = 0.02 Decreasing in Q with increase m.

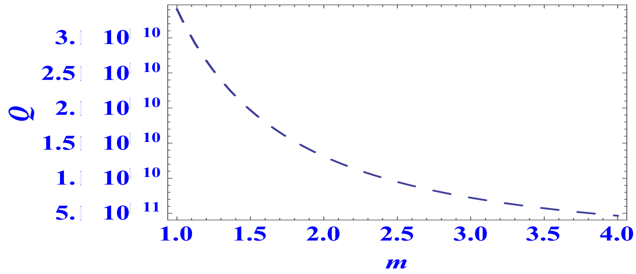

Fig. 4- Decrease of flow rate Q in the presence of magnetic field m.

Magnetic Viscosity of Viscous Fluid $\mu(m)$

Decreasing in flow rate leads to increasing in a magnetic viscosity for the viscous fluid under influence of the magnetic field.

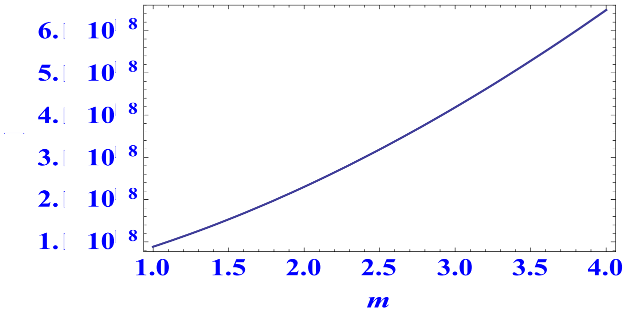

Fig. 5- Increasing of a magnetic viscosity μ with magnetic field m.

Results and Discussion of Blood

MHD flow of blood in narrow tubes is characterized in this model by several parameters: the dimensionless core radius α, the magnetic parameter m and the rheological parameters of the fluids, i.e., the viscosities of the core μ_i and outer layer μ_0.
The dimensionless core layer thickness α is chosen as α = 0.92, in some calculations, α is varied between 0 and 1. The viscosities of the core μ_i=4.03cp, and outer region μ_0=1.24cp.

Axial Velocity Distribution $w_i(\eta), w_0(\eta)$

Axial velocity profiles w_i and w_0, are represented graphically in Fig. 6 and 7 for two values of the pressure gradients, - dp/ dz = 76 dyne /mm³ and - dp/dz = 67.5 dyne / mm³. The effect of the magnetic field parameter m is also shown in that figures. The velocity profile of the core not parabolic but the bluntness parameter in Eq. (41) create the velocity profile of the core deviate from parabolic, when the magnetic field affect the bluntness decrease and the homogeneous fluid flow through a tube and these leads to speak about large vessel. The dimensionless core radius α is the important parameter affect on velocity profile,
when α decrease leads to bluntness decrease and the core become absent so the velocity of core region is the same of the velocity of outer region and the fluid become homogenous solution not suspension.

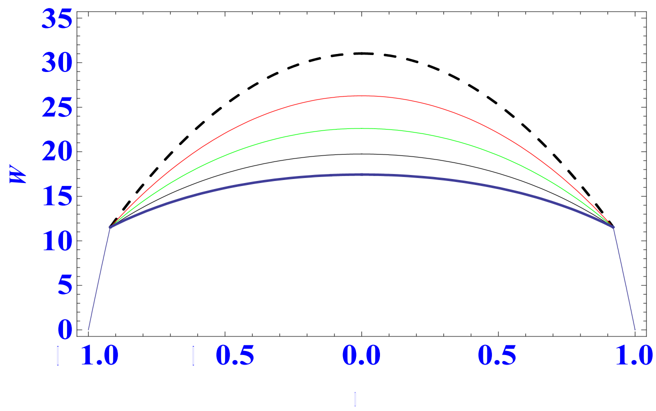

Fig. 6- Velocity profiles at α = 0.92, -dp/dz = 76dyne/mm³ and magnetic parameter (m = 0, 1, 2, 3, 4) (top to bottom).

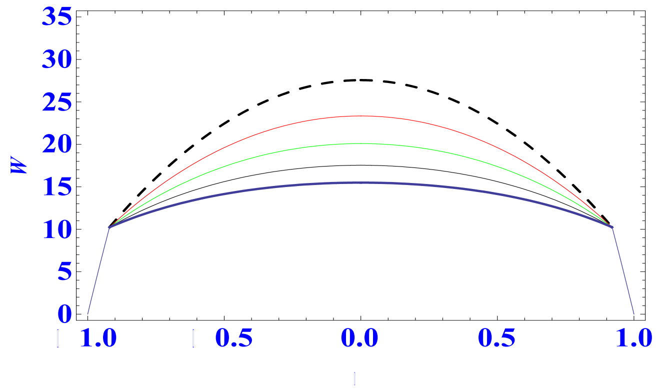

Fig. 7- Velocity profiles at α = 0.92, -dp/dz = 67.5dyne/mm3 and magnetic parameter (m = 0, 1, 2, 3, 4) (top to bottom).

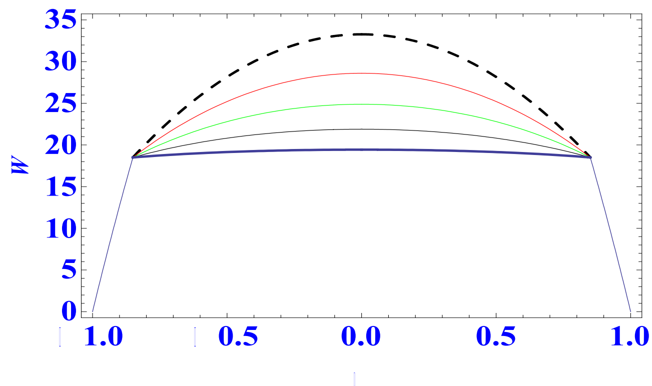

Fig. 8- Velocity profiles at α = 0.85, -dp/dz = 67.5dyne/mm3 and magnetic parameter (m = 0, 1, 2, 3, 4) (top to bottom).

Bluntness Distribution β

Parameter β in Eq. (41) is the bluntness velocity profile in the core region. For a given fluid, this parameter depends on the dimensionless core radius α, and magnetic parameter m. Fig. 9 shows that the bluntness parameter β increases as α increases. The increasing in β leads to the velocity profile become parabolic In other words. Fig. 10 shows that β decreases as m increases. Thus, the velocity profile becomes more parabolic when α is decrease or m is increase.

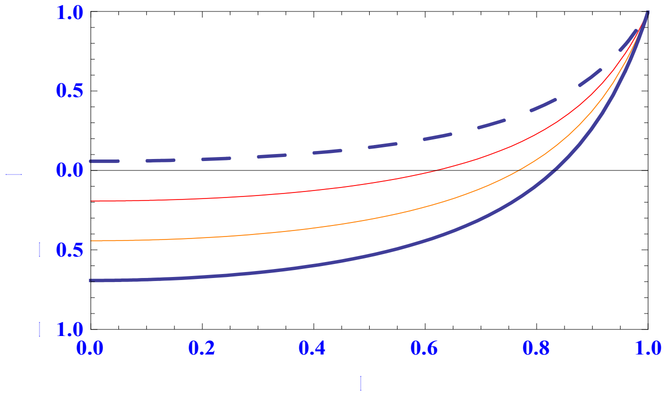

Fig. 9- Variation of bluntness parameter β with α at (m =1, 2, 3, 4) (top to bottom).

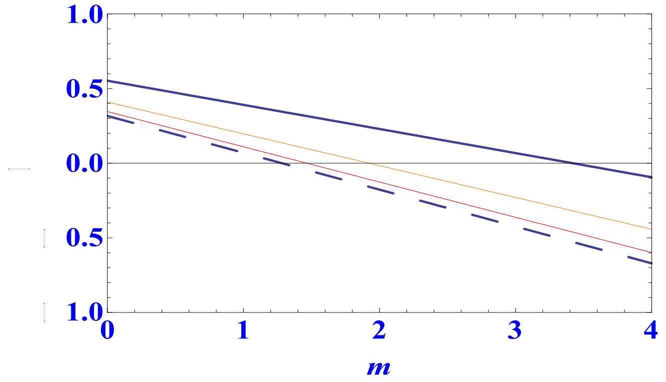

Fig. 10- Variation of bluntness parameter β with m at (α =0, 0.2, 0.4, 0.6, 0.8) (top to bottom).

Conclusion

In this paper, MHD incompresspole, electrical conducting viscous fluid in the tube is described. Governing differential equation which describes the flow in tube is described, and this equation is solved analytically. The effect of magnetic field parameter m on the velocity profile, flow rate and apparent magnetic viscosity is studied. It is found that increasing MHD parameter decreases the velocity and flow rate and this leads to increasing in apparent magnetic viscosity. The blood flow in the presence of a transverse magnetic field is taken as an example of this type of flow.

Magnetohydrodynamics (blood) flow in a narrow tube is described using two layered (two-phase) model. The model consists of a central core region enriched with various types of blood cells such as red blood cells (RBCs) and a cell free layer (plasma) surrounding the core.

Governing differential equations are solved analytically. The bluntness appears in velocity profile and this bluntness decrease by increasing magnetic field.

Therefore, a remarkable phenomenon is that by using an external magnetic field we can regulate the blood flow and treatment some of diseases such as bleeding and clotting.

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